Physics 8240: Advanced General Relativity (Fall 2023)

Kent Yagi

1 Basics

Instructor:	Kent Yagi
Office:	Astronomy 262
Lectures:	9:30–10:45am Tuesdays & Thursdays, New Cabell Hall 056
Office hours:	Mondays 3–4pm
(in my office)	Fridays 2–3pm
	(You are welcome to ask me questions via email at any time.)
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Grader:	Ben Wade (baw8td@virginia.edu)

Course Web Page: UVA Canvas *PHYS_8240-001* (23F Advanced General Relativity)

2 Course Objectives

The main goals of this course are to learn

1. geometrical aspects of General Relativity (in arbitrary bases)
2. initial value formulation
3. Lagrangian formulation
4. mass & angular momentum
5. black hole mechanics (especially stationary black holes and black hole perturbation)

I assume that you have taken intro GR course(s) or have self-studied intro-level GR. However, I plan to spend the first two weeks or so reviewing basic materials (and beyond) of GR from a differential geometry viewpoint.

3 Textbooks

I will mainly use the following (electronic versions available in UVA library):

- Eric Poisson [P], *A Relativist's Toolkit: The Mathematics of Black-Hole Mechanics*

This is a wonderful textbook that covers various advanced topics of GR in an accessible manner, with the aim towards understanding black hole mechanics.

I will also use the following

- Bernard Schutz [S], *Geometrical Methods of Mathematical Physics*

to review differential geometry and hopefully the following (if I have time)

- Michele Maggiore [M], *Gravitational Waves: Vol. 2, Astrophysics and Cosmology*

for black hole perturbation.

Other Books and References

1. Schutz, *A First Course in General Relativity*,
   (standard undergrad-level textbook, focuses on geometrical approach)

2. Carroll, *Spacetime and Geometry*,
   (standard grad-level textbook)

3. Misner, Thorne, and Wheeler, *Gravitation*
   (useful as a reference book),

4. Wald, *General Relativity*
   (advanced and mathematically rigorous),

5. Ferrari, Gualtieri, and Pani *General Relativity and Its Applications: Black Holes, Compact Stars and Gravitational Waves*
   (ample applications on black holes and gravitational waves),

6. Landau and Lifshitz, *The Classical Theory of Fields*
   (nice and concise),

7. Weinberg, *Gravitation and Cosmology*
   (non-geometric, field-theory approach),

8. Chandrasekhar, *The Mathematical Theory of Black Holes*
   (good reference book on black holes),

9. Frolov and Zelnikov, *Introduction to Black Hole Physics*
   (another detailed textbook on black hole physics),

10. Townsend, *Black holes: Lecture notes*
(nice lecture notes on black holes),

11. Maggiore, *Gravitational Waves: Vol. 1, Theory and Experiments* (excellent textbook covering various topics on gravitational waves),

12. Poisson and Will, *Gravity: Newtonian, post-Newtonian, Relativistic*
(detailed textbook on gravitational waves),

13. Creighton Gravitational-Wave Physics and Astronomy : An Introduction to Theory, Experiment and Data Analysis
(covers important topics in gravitational waves, including detectors and data analysis)

14. Sathyaprakash and Schutz Physics, Astrophysics and Cosmology with Gravitational Waves
(standard review article on gravitational waves)

4 Grade weighting

Grade will be determined with the following weighting and the final letter grade will be determined **on curves**:

50%	Homework
15%	Midterm exam
35%	Final exam

5 Lectures and Attendance

There is no strict attendance policy, though missing in-person classes may cause loss of learning. For example, I may explain things beyond what is in textbooks. You are responsible for the materials presented in class, turning in your homework on time (which you can also turn in online), and knowing any administrative announcements made, such as changes to the syllabus or changes to the scheduling of homework or exams.

6 Homework

I plan to assign HWs on the following Tuesdays:

1. Aug. 29
2. Sept. 5
3. Sept. 12
4. Sept. 19
5. Oct. 10

6. Oct. 17
7. Oct. 24
8. Oct. 31
9. Nov. 14
10. Nov. 28

You have one week to answer problems unless otherwise announced. Problems will be uploaded under "Assignments" in Canvas website. You may submit your answers either by bringing them to the classroom or submitting them electronically through "Assignments" in Canvas. Solutions will be uploaded under "Files" on the same website.

Mid-term and final exams will contain problems that are similar to homework problems, so take the latter seriously and make sure that you can solve them on your own.

Discussing the problems with each other is encouraged, but I expect each individual to write up their own solutions without direct copying. Copying another person's solution that you did not substantially participate in is unacceptable. **DO NOT** obtain or look at solution sets from previous years. The primary purpose of assigning these problems is for you to **struggle and learn**. Also, don't just write down answers, **show derivations**!

All the problems are designed so that you can do all the calculations with pen and paper. Do not use computational software tools like Mathematica for your first attempt (you may use them to check your calculations).

Your credit will be reduced by 20% over each 24-hour late submission. You may ask for an extension of due dates *in advance* provided you have good reasons to do so.

If you answer the course evaluation towards the end of the semester, your lowest HW will be dropped.

7 Exams

Exams to be held in the normal classroom.

- Midterm: Tuesday, October 10, 9:30 a.m. – 10:45 a.m.
- Final: Thursday, December 7, 2:00 p.m. – 5:00 p.m.

8 Topics

Here is a tentative plan on what topics to be covered, though they are subject to change and we may not have time to go through all of them.

1. Manifolds and Tensors [S: Sec. 2 and 3]
    - Manifolds

- Vectors
- One-forms and Tensors
- Lie Derivatives and Lie Drags

2. Riemannian Manifolds [S: Sec. 6]
    - Parallelism
    - Covariant Derivatives
    - Geodesics
    - Riemann Curvature
    - Metrics

3. Initial Value Formulation [P: Sec. 3]
    - Normal Vector
    - Induced Metric
    - Gauss-Stokes Theorem
    - Intrinsic Covariant Derivative
    - Extrinsic Curvature
    - Gauss-Codazzi Equations
    - 3+1 Decomposition
    - Initial Value Problem

4. Lagrangian Formulation [P: Sec. 4.1]
    - Field Theory
    - General Relativity

5. Gravitational Mass & Angular Momentum [P: Sec. 4.3]
    - ADM Mass & Angular Momentum
    - Killing Vectors and Symmetries
    - Komar Integrals

6. Static Black Holes [P: Sec. 5.1, 5.2, 5.4]
    - Frobenius Theorem
    - Kruskal and Penrose Diagrams
    - Apparent Horizon vs Event Horizon
    - Killing Horizon

- Surface Gravity
- Charged Black Hole

7. Stationary Black Holes [P: Sec. 5.3, 5.5]
    - Frame Dragging
    - Ergosphere
    - Event Horizon
    - Penrose Process
    - Singularities
    - Principal Null Congruences
    - Kruskal and Penrose Diagram
    - Smarr's Formula
    - Black Hole Mechanics & Thermodynamics

8. Black Hole Perturbations [M: Sec. 12]
    - Scalar Perturbations
    - Gravitational Perturbations: Static Black Holes
    - Quasinormal Modes
    - Newman-Penrose Formalism
    - Gravitational Perturbations: Stationary Black Holes

9 Special Accommodations

The Student Disability Access Center (SDAC) is available for you if you have any disabilities. Please work with them to discuss a range of options and how to request official accommodations.

I will make special accommodations related to health issues. If you let me know in advance, I will provide you a Zoom link so that you can attend the classes remotely. I will be happy to grant extensions on HWs if necessary.